100 numpy exercises

A joint effort of the numpy community

The goal is both to offer a quick reference for new and old users and to provide also a set of exercices for those who teach. If you remember having asked or answered a (short) problem, you can send a pull request. The format is:

```
#. Find indices of non-zero elements from [1,2,0,0,4,0]

   .. code:: python

      # Author: Somebody

      print(np.nonzero([1,2,0,0,4,0]))
```

Here is what the page looks like so far: http://www.labri.fr/perso/nrougier/teaching/numpy.100/index.html

Repository is at: https://github.com/rougier/numpy-100

Thanks to Michiaki Ariga, there is now a Julia version.

1. Import the numpy package under the name `np` (★☆☆)

```
import numpy as np
```

2. Print the numpy version and the configuration (★☆☆)

```
print(np.__version__)
np.show_config()
```

3. Create a null vector of size 10 (★☆☆)

```
Z = np.zeros(10)
print(Z)
```

4. How to get the documentation of the numpy add function from the command line? (★☆☆)

```
python -c "import numpy; numpy.info(numpy.add)"
```

5. Create a null vector of size 10 but the fifth value which is 1 (★☆☆)

```
Z = np.zeros(10)
Z[4] = 1
print(Z)
```

6. Create a vector with values ranging from 10 to 49 (★☆☆)

```
Z = np.arange(10,50)
print(Z)
```

7. Reverse a vector (first element becomes last) (★☆☆)

```
Z = np.arange(50)
Z = Z[::-1]
```

8. Create a 3x3 matrix with values ranging from 0 to 8 (★☆☆)

```
Z = np.arange(9).reshape(3,3)
print(Z)
```

9. Find indices of non-zero elements from [1,2,0,0,4,0] (★☆☆)

```
nz = np.nonzero([1,2,0,0,4,0])
print(nz)
```

10. Create a 3x3 identity matrix (★☆☆)

```
Z = np.eye(3)
print(Z)
```

11. Create a 3x3x3 array with random values (★☆☆)

```
Z = np.random.random((3,3,3))
print(Z)
```

12. Create a 10x10 array with random values and find the minimum and maximum values (★☆☆)

```
Z = np.random.random((10,10))
Zmin, Zmax = Z.min(), Z.max()
print(Zmin, Zmax)
```

13. Create a random vector of size 30 and find the mean value (★☆☆)

```
Z = np.random.random(30)
m = Z.mean()
print(m)
```

14. Create a 2d array with 1 on the border and 0 inside (★☆☆)

```
Z = np.ones((10,10))
Z[1:-1,1:-1] = 0
```

15. What is the result of the following expression? (★☆☆)

```
0 * np.nan
np.nan == np.nan
np.inf > np.nan
np.nan - np.nan
0.3 == 3 * 0.1
```

16. Create a 5x5 matrix with values 1,2,3,4 just below the diagonal (★☆☆)

```
Z = np.diag(1+np.arange(4),k=-1)
print(Z)
```

17. Create a 8x8 matrix and fill it with a checkerboard pattern (★☆☆)

```
Z = np.zeros((8,8),dtype=int)
Z[1::2,::2] = 1
Z[::2,1::2] = 1
print(Z)
```

18. Consider a (6,7,8) shape array, what is the index (x,y,z) of the 100th element?

```
print(np.unravel_index(100,(6,7,8)))
```

19. Create a checkerboard 8x8 matrix using the tile function (★☆☆)

```
Z = np.tile( np.array([[0,1],[1,0]]), (4,4))
print(Z)
```

20. Normalize a 5x5 random matrix (★☆☆)

```
Z = np.random.random((5,5))
Zmax, Zmin = Z.max(), Z.min()
Z = (Z - Zmin)/(Zmax - Zmin)
print(Z)
```

21. Create a custom dtype that describes a color as four unisgned bytes (RGBA) (★☆☆)

```
color = np.dtype([("r", np.ubyte, 1),
                  ("g", np.ubyte, 1),
                  ("b", np.ubyte, 1),
                  ("a", np.ubyte, 1)])
```

22. Multiply a 5x3 matrix by a 3x2 matrix (real matrix product) (★☆☆)

```
Z = np.dot(np.ones((5,3)), np.ones((3,2)))
print(Z)
```

23. Given a 1D array, negate all elements which are between 3 and 8, in place. (★☆☆)

```
# Author: Evgeni Burovski

Z = np.arange(11)
Z[(3 < Z) & (Z <= 8)] *= -1
```

24. What is the output of the following script? (★☆☆)

```
# Author: Jake VanderPlas

print(sum(range(5),-1))
from numpy import *
print(sum(range(5),-1))
```

25. Consider an integer vector Z, which of these expressions are legal? (★☆☆)

```
Z**Z
2 << Z >> 2
Z <- Z
1j*Z
Z/1/1
Z<Z>Z
```

26. What are the result of the following expressions?

```
np.array(0) // np.array(0)

np.array(0) // np.array(0.)
np.array(0) / np.array(0)
np.array(0) / np.array(0.)
```

27. How to round away from zero a float array ? (★☆☆)

```
# Author: Charles R Harris

Z = np.random.uniform(-10,+10,10)
print (np.trunc(Z + np.copysign(0.5, Z)))
```

28. Extract the integer part of a random array using 5 different methods (★★☆)

```
Z = np.random.uniform(0,10,10)

print (Z - Z%1)
print (np.floor(Z))
print (np.ceil(Z)-1)
print (Z.astype(int))
print (np.trunc(Z))
```

29. Create a 5x5 matrix with row values ranging from 0 to 4 (★★☆)

```
Z = np.zeros((5,5))
Z += np.arange(5)
print(Z)
```

30. Consider a generator function that generates 10 integers and use it to build an array (★☆☆)

```
def generate():
    for x in xrange(10):
        yield x
Z = np.fromiter(generate(),dtype=float,count=-1)
print(Z)
```

31. Create a vector of size 10 with values ranging from 0 to 1, both excluded (★★☆)

```
Z = np.linspace(0,1,12,endpoint=True)[1:-1]
print(Z)
```

32. Create a random vector of size 10 and sort it (★★☆)

```
Z = np.random.random(10)
Z.sort()
print(Z)
```

33. How to sum a small array faster than np.sum? (★★☆)

```
# Author: Evgeni Burovski

Z = np.arange(10)
np.add.reduce(Z)
```

34. Consider two random array A anb B, check if they are equal (★★☆)

```
A = np.random.randint(0,2,5)
B = np.random.randint(0,2,5)
equal = np.allclose(A,B)
print(equal)
```

35. Make an array immutable (read-only) (★★☆)

```
Z = np.zeros(10)
Z.flags.writeable = False
Z[0] = 1
```

36. Consider a random 10x2 matrix representing cartesian coordinates, convert them to polar coordinates (★★☆)

```
Z = np.random.random((10,2))
X,Y = Z[:,0], Z[:,1]
R = np.sqrt(X**2+Y**2)
T = np.arctan2(Y,X)
print(R)
print(T)
```

37. Create random vector of size 10 and replace the maximum value by 0 (★★☆)

```
Z = np.random.random(10)
Z[Z.argmax()] = 0
print(Z)
```

38. Create a structured array with `x` and `y` coordinates covering the [0,1]x[0,1] area (★★☆)

```
Z = np.zeros((10,10), [('x',float),('y',float)])
Z['x'], Z['y'] = np.meshgrid(np.linspace(0,1,10),
                             np.linspace(0,1,10))
print(Z)
```

39. Given two arrays, X and Y, construct the Cauchy matrix C (Cij = 1/(xi - yj))

```
# Author: Evgeni Burovski

X = np.arange(8)
Y = X + 0.5
C = 1.0 / np.subtract.outer(X, Y)
print(np.linalg.det(C))
```

40. Print the minimum and maximum representable value for each numpy scalar type (★★☆)

```
for dtype in [np.int8, np.int32, np.int64]:
   print(np.iinfo(dtype).min)
   print(np.iinfo(dtype).max)
for dtype in [np.float32, np.float64]:
   print(np.finfo(dtype).min)
   print(np.finfo(dtype).max)
   print(np.finfo(dtype).eps)
```

41. How to print all the values of an array? (★★☆)

```
np.set_printoptions(threshold=np.nan)
Z = np.zeros((25,25))
print(Z)
```

42. How to find the closest value (to a given scalar) in an array? (★★☆)

```
Z = np.arange(100)
v = np.random.uniform(0,100)
index = (np.abs(Z-v)).argmin()
print(Z[index])
```

43. Create a structured array representing a position (x,y) and a color (r,g,b) (★★☆)

```
 Z = np.zeros(10, [ ('position', [ ('x', float, 1),
                                   ('y', float, 1)]),
                    ('color',    [ ('r', float, 1),
                                   ('g', float, 1),
                                   ('b', float, 1)])])
print(Z)
```

44. Consider a random vector with shape (100,2) representing coordinates, find point by point distances (★★☆)

```
Z = np.random.random((10,2))
X,Y = np.atleast_2d(Z[:,0]), np.atleast_2d(Z[:,1])
D = np.sqrt( (X-X.T)**2 + (Y-Y.T)**2)
print(D)

# Much faster with scipy
import scipy
# Thanks Gavin Heverly-Coulson (#issue 1)
import scipy.spatial

Z = np.random.random((10,2))
D = scipy.spatial.distance.cdist(Z,Z)
print(D)
```

45. How to convert a float (32 bits) array into an integer (32 bits) in place?

```
Z = np.arange(10, dtype=np.int32)
Z = Z.astype(np.float32, copy=False)
```

46. How to read the following file? (★★☆)

```
# File content:
# -------------
1,2,3,4,5
6,,,7,8
,,9,10,11
# -------------
```

```
Z = np.genfromtxt("missing.dat", delimiter=",")
```

47. What is the equivalent of enumerate for numpy arrays? (★★☆)

```
Z = np.arange(9).reshape(3,3)
for index, value in np.ndenumerate(Z):
    print(index, value)
for index in np.ndindex(Z.shape):
    print(index, Z[index])
```

48. Generate a generic 2D Gaussian-like array (★★☆)

```
X, Y = np.meshgrid(np.linspace(-1,1,10), np.linspace(-1,1,10))
D = np.sqrt(X*X+Y*Y)
sigma, mu = 1.0, 0.0
G = np.exp(-( (D-mu)**2 / ( 2.0 * sigma**2 ) ) )
print(G)
```

49. How to randomly place p elements in a 2D array? (★★☆)

```
# Author: Divakar

n = 10
p = 3
Z = np.zeros((n,n))
np.put(Z, np.random.choice(range(n*n), p, replace=False),1)
```

50. Subtract the mean of each row of a matrix (★★☆)

```
# Author: Warren Weckesser

X = np.random.rand(5, 10)

# Recent versions of numpy
Y = X - X.mean(axis=1, keepdims=True)

# Older versions of numpy
Y = X - X.mean(axis=1).reshape(-1, 1)
```

51. How to I sort an array by the nth column? (★★☆)

```
# Author: Steve Tjoa

Z = np.random.randint(0,10,(3,3))
print(Z)
print(Z[Z[:,1].argsort()])
```

52. How to tell if a given 2D array has null columns? (★★☆)

```
# Author: Warren Weckesser

Z = np.random.randint(0,3,(3,10))
print((~Z.any(axis=0)).any())
```

53. Find the nearest value from a given value in an array (★★☆)

```
Z = np.random.uniform(0,1,10)
z = 0.5
m = Z.flat[np.abs(Z - z).argmin()]
print(m)
```

54. Create an array class that has a name attribute (★★☆)

```
class NamedArray(np.ndarray):
    def __new__(cls, array, name="no name"):
        obj = np.asarray(array).view(cls)
        obj.name = name
        return obj
    def __array_finalize__(self, obj):
        if obj is None: return
        self.info = getattr(obj, 'name', "no name")
```

```
Z = NamedArray(np.arange(10), "range_10")
print (Z.name)
```

55. Consider a given vector, how to add 1 to each element indexed by a second vector (be careful with repeated indices)? (★★★)

```
# Author: Brett Olsen

Z = np.ones(10)
I = np.random.randint(0,len(Z),20)
Z += np.bincount(I, minlength=len(Z))
print(Z)
```

56. How to accumulate elements of a vector (X) to an array (F) based on an index list (I)? (★★★)

```
# Author: Alan G Isaac

X = [1,2,3,4,5,6]
I = [1,3,9,3,4,1]
F = np.bincount(I,X)
print(F)
```

57. Considering a (w,h,3) image of (dtype=ubyte), compute the number of unique colors (★★★)

```
# Author: Nadav Horesh

w,h = 16,16
I = np.random.randint(0,2,(h,w,3)).astype(np.ubyte)
F = I[...,0]*256*256 + I[...,1]*256 +I[...,2]
n = len(np.unique(F))
print(np.unique(I))
```

58. Considering a four dimensions array, how to get sum over the last two axis at once? (★★★)

```
A = np.random.randint(0,10,(3,4,3,4))
sum = A.reshape(A.shape[:-2] + (-1,)).sum(axis=-1)
print(sum)
```

59. Considering a one-dimensional vector D, how to compute means of subsets of D using a vector S of same size describing subset indices? (★★★)

```
# Author: Jaime Fernández del Río

D = np.random.uniform(0,1,100)
S = np.random.randint(0,10,100)
D_sums = np.bincount(S, weights=D)
D_counts = np.bincount(S)
D_means = D_sums / D_counts
print(D_means)
```

60. How to get the diagonal of a dot product? (★★★)

```
# Author: Mathieu Blondel

# Slow version
np.diag(np.dot(A, B))

# Fast version
np.sum(A * B.T, axis=1)

# Faster version
np.einsum("ij,ji->i", A, B).
```

61. Consider the vector [1, 2, 3, 4, 5], how to build a new vector with 3 consecutive zeros interleaved between each value? (★★★)

```
# Author: Warren Weckesser
```

```
Z = np.array([1,2,3,4,5])
nz = 3
Z0 = np.zeros(len(Z) + (len(Z)-1)*(nz))
Z0[::nz+1] = Z
print(Z0)
```

62. Consider an array of dimension (5,5,3), how to mulitply it by an array with dimensions (5,5)? (★★★)

```
A = np.ones((5,5,3))
B = 2*np.ones((5,5))
print(A * B[:,:,None])
```

63. How to swap two rows of an array? (★★★)

```
# Author: Eelco Hoogendoorn

A = np.arange(25).reshape(5,5)
A[[0,1]] = A[[1,0]]
print(A)
```

64. Consider a set of 10 triplets describing 10 triangles (with shared vertices), find the set of unique line segments composing all the triangles (★★★)

```
# Author: Nicolas P. Rougier

faces = np.random.randint(0,100,(10,3))
F = np.roll(faces.repeat(2,axis=1),-1,axis=1)
F = F.reshape(len(F)*3,2)
F = np.sort(F,axis=1)
G = F.view( dtype=[('p0',F.dtype),('p1',F.dtype)] )
G = np.unique(G)
print(G)
```

65. Given an array C that is a bincount, how to produce an array A such that np.bincount(A) == C? (★★★)

```
# Author: Jaime Fernández del Río

C = np.bincount([1,1,2,3,4,4,6])
A = np.repeat(np.arange(len(C)), C)
print(A)
```

66. How to compute averages using a sliding window over an array? (★★★)

```
# Author: Jaime Fernández del Río

def moving_average(a, n=3) :
    ret = np.cumsum(a, dtype=float)
    ret[n:] = ret[n:] - ret[:-n]
    return ret[n - 1:] / n
Z = np.arange(20)
print(moving_average(Z, n=3))
```

67. Consider a one-dimensional array Z, build a two-dimensional array whose first row is (Z[0],Z[1],Z[2]) and each subsequent row is shifted by 1 (last row should be (Z[-3],Z[-2],Z[-1]) (★★★)

```
# Author: Joe Kington / Erik Rigtorp
from numpy.lib import stride_tricks

def rolling(a, window):
    shape = (a.size - window + 1, window)
    strides = (a.itemsize, a.itemsize)
    return stride_tricks.as_strided(a, shape=shape, strides=strides)
Z = rolling(np.arange(10), 3)
print(Z)
```

68. How to negate a boolean, or to change the sign of a float inplace? (★★★)

```
# Author: Nathaniel J. Smith
```

```
Z = np.random.randint(0,2,100)
np.logical_not(arr, out=arr)

Z = np.random.uniform(-1.0,1.0,100)
np.negative(arr, out=arr)
```

69. Consider 2 sets of points P0,P1 describing lines (2d) and a point p, how to compute distance from p to each line i (P0[i],P1[i])? (★★★)

```
def distance(P0, P1, p):
    T = P1 - P0
    L = (T**2).sum(axis=1)
    U = -((P0[:,0]-p[...,0])*T[:,0] + (P0[:,1]-p[...,1])*T[:,1]) / L
    U = U.reshape(len(U),1)
    D = P0 + U*T - p
    return np.sqrt((D**2).sum(axis=1))

P0 = np.random.uniform(-10,10,(10,2))
P1 = np.random.uniform(-10,10,(10,2))
p  = np.random.uniform(-10,10,( 1,2))
print(distance(P0, P1, p))
```

70. Consider 2 sets of points P0,P1 describing lines (2d) and a set of points P, how to compute distance from each point j (P[j]) to each line i (P0[i],P1[i])? (★★★)

```
# Author: Italmassov Kuanysh
# based on distance function from previous question
P0 = np.random.uniform(-10, 10, (10,2))
P1 = np.random.uniform(-10,10,(10,2))
p = np.random.uniform(-10, 10, (10,2))
print np.array([distance(P0,P1,p_i) for p_i in p])
```

71. Consider an arbitrary array, write a function that extract a subpart with a fixed shape and centered on a given element (pad with a `fill` value when necessary) (★★★)

```
# Author: Nicolas Rougier

Z = np.random.randint(0,10,(10,10))
shape = (5,5)
fill  = 0
position = (1,1)

R = np.ones(shape, dtype=Z.dtype)*fill
P  = np.array(list(position)).astype(int)
Rs = np.array(list(R.shape)).astype(int)
Zs = np.array(list(Z.shape)).astype(int)

R_start = np.zeros((len(shape),)).astype(int)
R_stop  = np.array(list(shape)).astype(int)
Z_start = (P-Rs//2)
Z_stop  = (P+Rs//2)+Rs%2

R_start = (R_start - np.minimum(Z_start,0)).tolist()
Z_start = (np.maximum(Z_start,0)).tolist()
R_stop = np.maximum(R_start, (R_stop - np.maximum(Z_stop-Zs,0))).tolist()
Z_stop = (np.minimum(Z_stop,Zs)).tolist()

r = [slice(start,stop) for start,stop in zip(R_start,R_stop)]
z = [slice(start,stop) for start,stop in zip(Z_start,Z_stop)]
R[r] = Z[z]
print(Z)
print(R)
```

72. Consider an array Z = [1,2,3,4,5,6,7,8,9,10,11,12,13,14], how to generate an array R = [[1,2,3,4], [2,3,4,5], [3,4,5,6], ..., [11,12,13,14]]? (★★★)

```
# Author: Stefan van der Walt

Z = np.arange(1,15,dtype=uint32)
R = stride_tricks.as_strided(Z,(11,4),(4,4))
print(R)
```

73. Compute a matrix rank (★★★)

```
# Author: Stefan van der Walt

Z = np.random.uniform(0,1,(10,10))
U, S, V = np.linalg.svd(Z) # Singular Value Decomposition
rank = np.sum(S > 1e-10)
```

74. How to find the most frequent value in an array?

```
Z = np.random.randint(0,10,50)
print(np.bincount(Z).argmax())
```

75. Extract all the contiguous 3x3 blocks from a random 10x10 matrix (★★★)

```
# Author: Chris Barker

Z = np.random.randint(0,5,(10,10))
n = 3
i = 1 + (Z.shape[0]-3)
j = 1 + (Z.shape[1]-3)
C = stride_tricks.as_strided(Z, shape=(i, j, n, n), strides=Z.strides + Z.strides)
print(C)
```

76. Create a 2D array subclass such that Z[i,j] == Z[j,i] (★★★)

```
# Author: Eric O. Lebigot
# Note: only works for 2d array and value setting using indices

class Symetric(np.ndarray):
    def __setitem__(self, (i,j), value):
        super(Symetric, self).__setitem__((i,j), value)
        super(Symetric, self).__setitem__((j,i), value)

def symetric(Z):
    return np.asarray(Z + Z.T - np.diag(Z.diagonal())).view(Symetric)

S = symetric(np.random.randint(0,10,(5,5)))
S[2,3] = 42
print(S)
```

77. Consider a set of p matrices wich shape (n,n) and a set of p vectors with shape (n,1). How to compute the sum of of the p matrix products at once? (result has shape (n,1)) (★★★)

```
# Author: Stefan van der Walt

p, n = 10, 20
M = np.ones((p,n,n))
V = np.ones((p,n,1))
S = np.tensordot(M, V, axes=[[0, 2], [0, 1]])
print(S)

# It works, because:
# M is (p,n,n)
# V is (p,n,1)
# Thus, summing over the paired axes 0 and 0 (of M and V independently),
# and 2 and 1, to remain with a (n,1) vector.
```

78. Consider a 16x16 array, how to get the block-sum (block size is 4x4)? (★★★)

```
# Author: Robert Kern

Z = np.ones(16,16)
k = 4
S = np.add.reduceat(np.add.reduceat(Z, np.arange(0, Z.shape[0], k), axis=0),
                                       np.arange(0, Z.shape[1], k), axis=1)
```

79. How to implement the Game of Life using numpy arrays? (★★★)

```
# Author: Nicolas Rougier

def iterate(Z):
```

```
    # Count neighbours
    N = (Z[0:-2,0:-2] + Z[0:-2,1:-1] + Z[0:-2,2:] +
         Z[1:-1,0:-2]                + Z[1:-1,2:] +
         Z[2:  ,0:-2] + Z[2:  ,1:-1] + Z[2:  ,2:])

    # Apply rules
    birth = (N==3) & (Z[1:-1,1:-1]==0)
    survive = ((N==2) | (N==3)) & (Z[1:-1,1:-1]==1)
    Z[...] = 0
    Z[1:-1,1:-1][birth | survive] = 1
    return Z

Z = np.random.randint(0,2,(50,50))
for i in range(100): Z = iterate(Z)
```

80. How to get the n largest values of an array (★★★)

```
Z = np.arange(10000)
np.random.shuffle(Z)
n = 5

# Slow
print (Z[np.argsort(Z)[-n:]])

# Fast
print (Z[np.argpartition(-Z,n)[:n]])
```

81. Given an arbitrary number of vectors, build the cartesian product (every combinations of every item) (★★★)

```
# Author: Stefan Van der Walt

def cartesian(arrays):
    arrays = [np.asarray(a) for a in arrays]
    shape = (len(x) for x in arrays)

    ix = np.indices(shape, dtype=int)
    ix = ix.reshape(len(arrays), -1).T

    for n, arr in enumerate(arrays):
        ix[:, n] = arrays[n][ix[:, n]]

    return ix

print (cartesian(([1, 2, 3], [4, 5], [6, 7])))
```

82. How to create a record array from a regular array? (★★★)

```
Z = np.array([("Hello", 2.5, 3),
              ("World", 3.6, 2)])
R = np.core.records.fromarrays(Z.T,
                               names='col1, col2, col3',
                               formats = 'S8, f8, i8')
```

83. Consider a large vector Z, compute Z to the power of 3 using 3 different methods (★★★)

```
Author: Ryan G.

x = np.random.rand(5e7)

%timeit np.power(x,3)
1 loops, best of 3: 574 ms per loop

%timeit x*x*x
1 loops, best of 3: 429 ms per loop

%timeit np.einsum('i,i,i->i',x,x,x)
1 loops, best of 3: 244 ms per loop
```

84. Consider two arrays A and B of shape (8,3) and (2,2). How to find rows of A that contain elements of each row of B regardless of the order of the elements in B? (★★★)

```
# Author: Gabe Schwartz
```

```
A = np.random.randint(0,5,(8,3))
B = np.random.randint(0,5,(2,2))

C = (A[..., np.newaxis, np.newaxis] == B)
rows = (C.sum(axis=(1,2,3)) >= B.shape[1]).nonzero()[0]
print(rows)
```

85. Considering a 10x3 matrix, extract rows with unequal values (e.g. [2,2,3]) (★★★)

```
# Author: Robert Kern

Z = np.random.randint(0,5,(10,3))
E = np.logical_and.reduce(Z[:,1:] == Z[:,:-1], axis=1)
U = Z[~E]
print(Z)
print(U)
```

86. Convert a vector of ints into a matrix binary representation (★★★)

```
# Author: Warren Weckesser

I = np.array([0, 1, 2, 3, 15, 16, 32, 64, 128])
B = ((I.reshape(-1,1) & (2**np.arange(8))) != 0).astype(int)
print(B[:,::-1])

# Author: Daniel T. McDonald

I = np.array([0, 1, 2, 3, 15, 16, 32, 64, 128], dtype=np.uint8)
print(np.unpackbits(I[:, np.newaxis], axis=1))
```

87. Given a two dimensional array, how to extract unique rows? (★★★)

```
# Author: Jaime Fernández del Río

Z = np.random.randint(0,2,(6,3))
T = np.ascontiguousarray(Z).view(np.dtype((np.void, Z.dtype.itemsize * Z.shape[1])))
_, idx = np.unique(T, return_index=True)
uZ = Z[idx]
print(uZ)
```

88. Considering 2 vectors A & B, write the einsum equivalent of inner, outer, sum, and mul function (★★★)

```
# Author: Alex Riley
# Make sure to read: http://ajcr.net/Basic-guide-to-einsum/

np.einsum('i->', A)       # np.sum(A)
np.einsum('i,i->i', A, B) # A * B
np.einsum('i,i', A, B)    # np.inner(A, B)
np.einsum('i,j', A, B)    # np.outer(A, B)
```

89. Considering a path described by two vectors (X,Y), how to sample it using equidistant samples (★★★)?

```
# Author: Bas Swinckels

phi = np.arange(0, 10*np.pi, 0.1)
a = 1
x = a*phi*np.cos(phi)
y = a*phi*np.sin(phi)

dr = (np.diff(x)**2 + np.diff(y)**2)**.5 # segment lengths
r = np.zeros_like(x)
r[1:] = np.cumsum(dr)                # integrate path
r_int = np.linspace(0, r.max(), 200) # regular spaced path
x_int = np.interp(r_int, r, x)       # integrate path
y_int = np.interp(r_int, r, y)
```

90. Given an integer n and a 2D array X, select from X the rows which can be interpreted as draws from a multinomial distribution with n degrees, i.e., the rows which only contain integers and which sum to n. (★★★)

```
# Author: Evgeni Burovski

X = np.asarray([[1.0, 0.0, 3.0, 8.0],
                [2.0, 0.0, 1.0, 1.0],
                [1.5, 2.5, 1.0, 0.0]])
n = 4
M = np.logical_and.reduce(np.mod(X, 1) == 0, axis=-1)
M &= (X.sum(axis=-1) == n)
print(X[M])
```